AIAA JOURNAL
Vol. 36, No. 1, January 1998

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscr ipt pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Two-Equation Turbulence Closure Model for Wall Bounded and Free Shear Flows

David F. Robinson* and H. A. Hassan†
North Carolina State University,
Raleigh, North Carolina 27695-7910

Introduction

EXISTING linear k–ω and k–ε turbulence closure models are incapable of describing wall bounded and free shear flows using one set of model constants and boundary conditions. It is generally agreed that the problem with existing models comes from the length scale determining equation ε or ω. The reason for this can be traced to the fact that these equations were developed for high-turbulence Reynolds numbers Re_t but are being employed in situations where turbulence Reynolds number is low.

For most turbulent shear flows, the turbulence Reynolds number is typically large enough that one can assume the small scales to be nearly independent of the large scales and, thus, their dissipation rate is isotropic. If one makes this isotropy assumption, the terms in the exact dissipation equation that depend on the mean flow can be neglected. However, for most shear flows, the turbulence Reynolds number varies from very large values in the bulk of the flow to very small values near walls and outside of the boundary layer. For regions of low turbulence Reynolds number, the small scales of turbulence are weakly dependent on the large scales and, therefore, on the mean flow. Because of this, terms appearing in the exact dissipation equation that depend on the mean flow cannot be neglected.

The preceding considerations were the basis of a new two-equation model.[1] Instead of modeling the exact equation for dissipation, attention was focused on the enstrophy or the variance of vorticity equation.[2] The model developed in Ref. 1 was used to describe free shear layers (wakes, jets, and mixing layers). A single set of model constants was used in all calculations (Table 1). Excellent predictions of growth rates, shear stress, and velocity distributions were obtained. The object of this work is to use the same set of model constants developed in Ref. 1 to study wall bounded flows. The implementation of the model will proceed in two stages. In the first, the model is implemented in a boundary-layer code,[3] and the results are illustrated by calculating a flow past a flat plate and its wake. This calculation, which is relevant to the calculation of multielement airfoils, was attempted earlier using a k–ε model[4,5]; however, predictions failed to reproduce correct growth rates. In the second, the model is implemented in a Navier–Stokes code[6] and the results are illustrated by calculating attached flows past airfoils.

Received April 26, 1996; presented as Paper 96-2057 at the AIAA 27th Fluid Dynamics Conference, New Orleans, LA, June 17–20, 1996; revision received July 24, 1997; accepted for publication Oct. 3, 1997.

*Research Assistant, Mechanical and Aerospace Engineering. Student Member AIAA.

†Professor, Mechanical and Aerospace Engineering. Associate Fellow AIAA.

Analysis

The k and ζ equations used are the same as those used in Ref. 1. Two adjustments were made in the model to accommodate near-wall behavior. The first pertains to the dissipation term in the ζ equation. The turbulence timescale $k/\nu\zeta$ cannot be less than Kolmogorov's timescale. To enforce this, the dissipation term now has the form

$$\frac{\beta_5 \zeta^{\frac{3}{2}}}{R_k + \delta} \tag{1}$$

For the results presented here, $\delta = 0.1$.

Under high-turbulence Reynolds number conditions, the eddy viscosity is given as

$$\nu_t = C_\mu k^2/\nu\zeta, \qquad C_\mu = 0.09 \tag{2}$$

In wall bounded flows, the eddy viscosity and the turbulence Reynolds number are both zero at the wall; therefore, the preceding expression is traditionally multiplied by the function f_μ. There is no unanimity on the form of f_μ in the near-wall region.[7] Recent implementations such as that of Speziale et al.[8] employ a function that depends on both turbulence Reynolds number and y^+, where

$$Re_t = k^2/\nu^2\zeta, \qquad y^+ = yU_\tau/\nu, \qquad U_\tau = \sqrt{\tau_w/\rho} \tag{3}$$

where τ_w is the wall shearing stress. Because separation may take place in the presence of adverse pressure gradients, a different representation that does not require y^+ is employed. The resulting expression is given as

$$f_\mu = \min\left(f_\mu^*, 1.0\right), \qquad f_\mu^* = \left(1 + \frac{C_{\mu 1}}{Re_t^{\frac{3}{2}}}\right) \exp\left(\frac{-\sqrt{ky}}{\nu C_{\mu 2}}\right) \tag{4}$$

where

$$C_{\mu 1} = 4.0, \qquad C_{\mu 2} = 40.0 \tag{5}$$

The boundary conditions for the two-equation model are

$$k_w = 0, \qquad k_\infty = \tfrac{3}{2}(\mathcal{T}U_\infty)^2 \tag{6}$$

and

$$\left.\frac{\partial k}{\partial y}\right|_w = 0 \tag{7}$$

where $\mathcal{T}$ is the turbulent intensity ($\approx$1%) and ζ_∞ is determined by specifying a freestream ratio of μ_t/μ along with k_∞. The term ζ has no physical boundary condition, and Eq. (7), in conjunction with the k equation, will be used to determine ζ_w. From the k equation[1]

$$\nu_w \zeta_w = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}y}\left(\nu\frac{\mathrm{d}k}{\mathrm{d}y}\right)_w = \left(\nu\frac{\partial k}{\partial y}\right)_1 \Big/ 3\Delta y_1 = \frac{\nu_1(k_2 - k_1)}{3\Delta y_1 \Delta y_2} \tag{8}$$

with

$$\Delta y_1 = y_1 - y_w, \qquad \Delta y_2 = y_2 - y_1$$

In the preceding equations, the subscripts ∞ and w designate freestream and wall, respectively, and 1 and 2 designate the first and second points off the wall.

Table 1 Model constants

Constants	C_μ	κ	α_3	β_4	β_5	β_6	β_7	β_8	σ_τ	$1/\sigma_k$	$1/\sigma_\zeta$
k–ζ	0.09	0.40	0.35	0.42	2.37	0.10	0.75	2.30	2.00	1.80	1.46

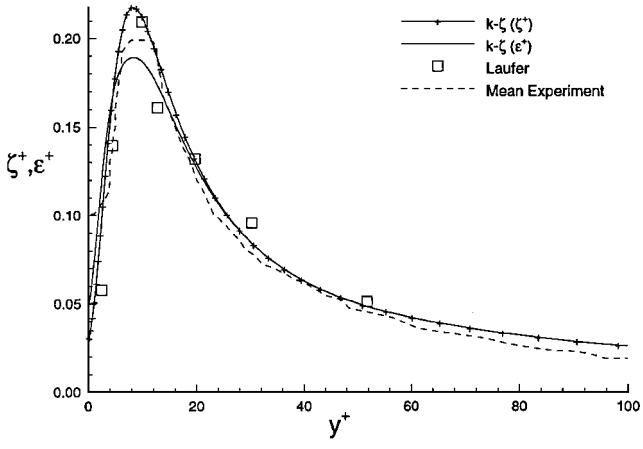

Fig. 1 Near-wall behavior of enstrophy and dissipation rate.

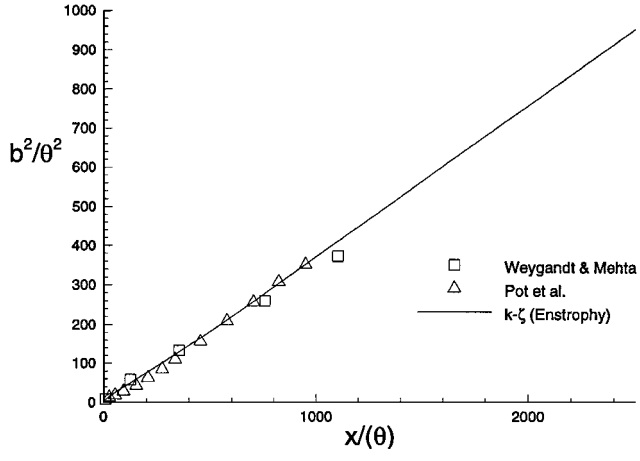

Fig. 2 Streamwise variation of wake half-width.

Results

There are a number of tests a turbulence model must meet to be considered a successful model. First and foremost, it must predict the correct skin-friction and pressure coefficients and other near-wall measurements[7] of shearing stress, kinetic energy, and dissipation. Second, it must predict the correct growth rate, asymptotic shearing stress distribution, and velocity in the far wake. Finally, it must predict the correct B constant in the expression for the velocity in the log-law region and the manner in which k varies with y in the near-wall region (see Table 4.5 in Ref. 9).

Only limited results are presented here, with other results available in Ref. 10. Results obtained using a boundary-layer code used between 101 and 201 points in the y direction. Results presented here, which are grid resolved, employed 201 points. In general, good agreement is indicated with experiment and direct numerical simulation (DNS).[11] The only exception is shown in Fig. 1, which shows a plot of ζ^+ and ε^+ vs y^+ in the near-wall region where

$$\zeta^+ = \zeta\nu^2/U_\tau^4, \qquad \varepsilon^+ = \varepsilon\nu/U_\tau^4, \qquad y^+ = U_\tau y/\nu \tag{9}$$

Also shown is the average of available data[7] together with Laufer's[12] experimental measurements. The DNS data show that the maximum of ε^+ occurs at the wall. It is not clear which modeled term is responsible for the indicated behavior; in all probability a collection

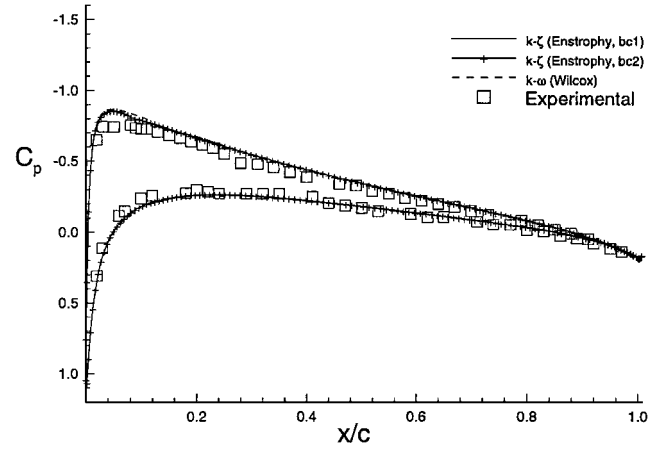

Fig. 3 Pressure distribution for NACA 0012 airfoil: $M_\infty = 0.502$, $Re_\infty = 2.91 \times 10^6$, and $\alpha = 2.06$.

of terms is responsible for the discrepancy. Note that the relation between ζ and ε follows from the k equation[10] as

$$\varepsilon = \nu\zeta + \nu\frac{\partial^2 k}{\partial x_i \partial x_i} + \nu\varepsilon_{ikm}\frac{\partial}{\partial x_k}(\overline{u_i'\omega_m'})$$

$$\simeq \nu\zeta + \frac{2}{3}\nu\frac{\mathrm{d}^2 k}{\mathrm{d}y^2} \tag{10}$$

where the approximation is for the near-wall region.

In general, good agreement with experiment is indicated in the wake region.[10] Typical of the results are those shown in Fig. 2, which shows the wake half-width b as a function of x/θ, where θ is the momentum thickness. The experimental results were obtained from Pot[13] and from Weygandt and Mehta.[14]

The final comparison (Fig. 3) shows a Navier–Stokes calculation of pressure coefficient C_p for a NACA 0012 airfoil at a Mach number of 0.502 and an angle of attack of 2.06 deg. The experimental results are obtained from Ref. 15. Grid convergence studies were conducted. The results shown employed a grid of 301×101. Again, good agreement with experiment is indicated.

Conclusion

In conclusion, it has been shown that the k–ζ model can be used for both wall bounded and free shear flows using one set of model constants. The model correctly predicts the spreading rate of a wake, a result that is important for calculations involving high-lift devices. Further, the same model can be incorporated into either a boundary-layer or a Navier–Stokes code.

Acknowledgments

This work is supported in part by NASA Grant NAG-1-244. The authors would like to express appreciation to the North Carolina Supercomputing Center for use of the facilities.

References

[1]Robinson, D. F., Harris, J. E., and Hassan, H. A., "Unified Turbulence Closure Model for Axisymmetric and Planar Free Shear Flows," *AIAA Journal*, Vol. 33, No. 12, 1995, pp. 2324–2331.

[2]Tennekes, H., and Lumley, J. L., *A First Course in Turbulence*, MIT Press, Cambridge, MA, 1989.

[3]Harris, J. E., and Blanchard, D. K., "Computer Program for Solving Laminar, Transitional, or Turbulent Compressible Boundary-Layer Equations for Two-Dimensional and Axisymmetric Flow," NASA TM 83207, Feb. 1982.

[4]Patel, V. C., and Scheuerer, G., "Calculation of Two-Dimensional Near and Far Wakes," *AIAA Journal*, Vol. 20, No. 7, 1982, pp. 900–907.

[5]Patel, V. C., and Chen, H. C., "Turbulent Wake of a Flat Plate," *AIAA Journal*, Vol. 25, No. 8, 1987, pp. 1078–1085.

[6]Gaffney, R. L., Salas, M. D., and Hassan, H. A., "An Abbreviated Reynolds Stress Turbulence Model for Airfoil Flows," AIAA Paper 90-1468, June 1990.
[7]Patel, V. C., Rodi, W., and Scheuerer, G., "Turbulence Models for Near-Wall and Low Reynolds Number Flows: A Review," *AIAA Journal*, Vol. 23, No. 9, 1985, pp. 1308–1319.
[8]Speziale, C. G., Abid, R., and Anderson, E. C., "Critical Evaluation of Two-Equation Models for Near-Wall Turbulence," *AIAA Journal*, Vol. 30, No. 2, 1992, pp. 324–331.
[9]Wilcox, D. C., *Turbulence Modeling for CFD*, DCW Industries, La Cañada, CA, 1993.
[10]Robinson, D. F., and Hassan, H. A., "A Two-Equation Turbulence Closure Model for Wall Bounded and Free Shear Flows," AIAA Paper 96-2057, June 1996.
[11]Mansour, N. N., Kim, J., and Moin, P., "Reynolds Stress and Dissipation Rate Budgets in Turbulent Shear Flows," *Journal of Fluid Mechanics*, Vol. 194, Pt. 1, 1988, pp. 15–44.
[12]Laufer, J., "Investigation of Turbulent Shear Flow in a Two-Dimensional Channel," NACA Rept. 1053, 1951.
[13]Pot, P. J., "Measurement in a Two-Dimensional Wake and in a Two-Dimensional Wake Merging into a Boundary Layer," NLR, TR-79063 U, The Netherlands, 1979.
[14]Weygandt, J. H., and Mehta, R. D., "Asymptotic Behavior of a Flat Plate Wake," NASA CR-185917, 1989.
[15]Thibert, J. J., Granjacques, M., and Ohman, L. H., "Experimental Data Base for Computer Program Assessment, NACA 0012 Airfoil," AGARD Advisory Rept. No. 138, May 1979.

D. S. McRae
Associate Editor

Alternative to the e^n Method for Determining Onset of Transition

E. W. Warren* and H. A. Hassan†
North Carolina State University, Raleigh, North Carolina 27695-7910

Introduction

THE process of transition from laminar to turbulent flow remains one of the most important unsolved problems in fluid mechanics and aerodynamics. Transitional flows are characterized by increased skin friction and heat transfer, and the accurate determination of heating rates and drag critically depends on the ability to predict the onset and extent of transition. However, no mathematical model exists that can accurately predict the location of transition under a wide range of conditions. Design engineers resort to methods that are based on either empirical correlations or linear stability theory.

The e^n method is currently the method of choice for determining transition onset. The method is based on linear stability theory and generally requires the following steps.

1) Mean flow must be precalculated at a large number of streamwise locations along the body of interest.

2) At each streamwise station, a local linear stability analysis is performed. By assumptions of the linear theory, the unsteady disturbances are decomposed into separate normal modes of different frequency. The stability equations are solved for the spatial amplification rate of each unstable frequency.

3) An amplitude ratio for each frequency is then calculated by integrating the spatial amplification rate in the streamwise direction on the body, i.e.,

$$\ln\left(\frac{A}{A_0}\right) = \int_{x_0}^{x} \alpha_i \, \mathrm{d}x \tag{1}$$

4) The n factor is then determined by taking the maximum of the just calculated quantity at each streamwise location.

The major problem with the e^n method is that the n factor does not represent the amplitude of a disturbance in the boundary layer but rather an amplification factor from an unknown amplitude A_0. The amplitude A_0 represents the amplitude of a disturbance of specified frequency at its neutral stability point. Its value is related to the external disturbance environment through some generally unknown receptivity process. As a consequence, the value of n that determines transition onset must be correlated to available experimental data.[1] Additionally, the e^n method requires the use of several computational tools such as a boundary-layer or Navier–Stokes flow solver to calculate the mean flow and the linear stability solver to determine the amplification rates.[2] Methods based on the nonlinear parabolized stability equations[3] (PSE) are being used to determine transition onset but they have not received the wide acceptance enjoyed by the e^n method. Methods based on the PSE also require precalculation of the mean flow and specification of initial conditions such as frequency and disturbance eigenfunctions. Methods based on linear stability theory only provide an estimation of the location of transition and can provide no information about the subsequent transitional and turbulent flow.

In this work, a different approach has been developed, which does not require precalculation of the mean flow or the specification of frequencies. It determines the transition onset and calculates the laminar, transitional, and turbulent regions in a single computation. The approach employs a two-equation model similar to that employed in turbulent calculations. It is based on the premise that, if a flow quantity can be written as the sum of a mean and a fluctuating quantity, then the exact equations that govern the fluctuations and their averages are identical irrespective of the nature of the oscillations, i.e., laminar, transitional, or turbulent. Moreover, if it is possible to model the equations governing the mean energy of the fluctuations and their rate of decay (or other equations) in such a way that one does not appeal to their nature, then the resulting model equations will be formally identical. However, the parameters that appear in the modeled equations will depend on the nature of the fluctuations. As an illustration, let us assume that we employ a Boussinesq approximation to model the stresses resulting from the fluctuations, i.e.,

$$\tau_{ij} = -\overline{\rho u_i' u_j'} = \mu_t \left(2S_{ij} - \frac{2}{3}\delta_{ij}\frac{\partial U_m}{\partial x_m}\right) - \frac{2}{3}\rho k \delta_{ij} \tag{2}$$

where

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right), \qquad k = \frac{1}{2}\overline{u_i' u_i'}$$

and where ρ is the density, U_i is the mean velocity, δ_{ij} is the Kronecker delta, and μ_t is the coefficient of viscosity brought about by the presence of fluctuations. The form indicated in Eq. (2) is used for all fluctuations, but the expressions for μ_t are quite different because the physics governing them is different.

The present approach is developed in conjunction with the k–ζ turbulence model of Robinson et al.[4] Details of the approach are given in Ref. 5. To explain its nature, the modeled k-equation is written as[5]

$$\frac{\mathrm{D}k}{\mathrm{D}t} = -\overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j} - \frac{k}{\tau_k} + \frac{\partial}{\partial x_j}\left[\left(\frac{\nu}{3} + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{3}$$

where $\nu_t = \mu_t/\rho$ and ν is the molecular kinematic viscosity. As may be seen from Eq. (3) and Refs. 5 and 6, to close the model one needs

Received Nov. 16, 1996; revision received Sept. 5, 1997; accepted for publication Sept. 10, 1997.

*Research Assistant, Mechanical and Aerospace Engineering. Member AIAA.
†Professor, Mechanical and Aerospace Engineering. Associate Fellow, AIAA.